MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2022/2023 SESSION

PPS0016 – INTRODUCTION TO PROBABILITY AND STATISTICS

(All sections / Groups)

4 OCTOBER 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 6 pages (excluding the cover page) with 5 questions.
2. Answer all questions.
3. Unless stated otherwise, if an answer is given as a decimal, it should be rounded to four significant figures.
4. Write your answers in the Answer Booklet provided.
5. Show all relevant steps to obtain maximum marks.

Question 1 [20 marks]

(a) Ali wants to find out the proportion of people who have hearing loss. He randomly selects 200 people and tests their hearing where the threshold of their hearing is measured in decibels. A person has normal hearing if they can hear at a threshold of 20 dB or better. Else, they have hearing loss. He finds out that 4% of them have hearing loss.

(i) Fill in the table below by stating or describing each term according to the above passage. [6 marks]

Population	
Individual	
Sample	
Statistic	
Parameter	
Variable	

(ii) Which group the variable you state in (a)(i) is in: qualitative, quantitative, discrete, continuous? Justify your answer. [4 marks]

(b) The frequency distribution below shows a sample of time taken by a mechanic to fix a car sent to the workshop.

Time (minutes)	Frequency
11–20	3
21–30	8
31–40	13
41–50	13
51–60	13

(i) Copy the below table and complete it. [6 marks]

Time (minutes)	Frequency	Midpoint	Midpoint × Frequency	Cumulative frequency
11–20	3			
21–30	8			
31–40	13			
41–50	13			
51–60	13			
Total				

(ii) Half of the cars take less than k minutes to be fixed. Find k. [2 marks]

(iii) In general, each car takes p minutes to be fixed. Find p. [2 marks]

Continued...

Question 2 [20 marks]

(a) In a survey of 50 people on the fruits that they like, the results are as follows:

22 like rambutan

21 like durian

25 like mangosteen

12 like both rambutan and mangosteen

8 like both rambutan and durian

6 like all the three fruits

5 do not like the three fruits

(i) Draw a Venn diagram to visualize the above information. Use R, D, and M as the events that a person likes rambutan, durian, and mangosteen respectively. Then, find the number of people who like mangosteen only. [6 marks]

(ii) Find the probability that a randomly selected person likes at most two fruits only. [2 marks]

(iii) Given that a person likes only one fruit, find the probability that this person likes rambutan. [2 marks]

(b) The probability that you will bring an umbrella when going out is 0.1. If you bring an umbrella, the probability that it will rain later on is 0.6 whereas if you don't bring one, the probability that it will rain later on is 0.3. Let U be the event that you will bring an umbrella when going out, and R the event that it will rain later on.

(i) Draw a tree diagram to represent the given information. No need to show the leaves. [6 marks]

(ii) Find the probability that it will rain later on. [2 marks]

(iii) Given that it rains later on, find the probability that you didn't bring an umbrella. [2 marks]

Continued...

Question 3 [20 marks]

(a) (i) Show that the function $f(x)=\frac{x}{15}$ for $x=1,2,3,4,5$ is a valid probability mass function for the discrete random variable X. [4½ marks]

(ii) Construct the cumulative distribution function of X in tabular form. [4½ marks]

(b) A random variable X has probability density function given by:

$$f(x)=\begin{cases}\frac{1}{3}, & 0\le x\le 2\\ \frac{1}{19}x^2, & 2<x\le 3\\ 0, & \text{otherwise}\end{cases}$$

(i) Show that $f(x)$ is a valid probability density function. [5 marks]

(ii) Find and write the cumulative distribution function $F(x)$ as a piecewise function. [6 marks]

Continued...

Question 4 [20 marks]

(a) State the four conditions of a binomial experiment. [4 marks]

(b) The probability that a machine fails on a particular day is 2.5% and follows a binomial probability distribution. A factory has 8 such machines.

(i) Find the probability that at most 2 machines fail on a particular day. [3 marks]

(ii) On average, how many machines are working on a particular day? [2 marks]

(c) The number of fish caught by Ali in a pond follows a Poisson probability distribution, and averages at 2 fish per 3 hours.

(i) What is the probability that Ali will get 3 fish if he spends 3 hours fishing? [2 marks]

(ii) Ali will spend 2 hours fishing tomorrow. What is the probability that he will get more than 2 fish? [4 marks]

(iii) Ali will spend 2 hours each day for fishing next week. What is the mean number of fish that he will catch next week? [2 marks]

(d) Z is a standard normal random variable. State whether the following statement is true or false. [3 marks]

(i) $P(Z < -0.44) = P(Z > 0.44)$

(ii) $P(Z > -2.1) = P(Z < 2.1)$

(iii) $P(Z > 2.1) = 1 - P(Z < 2.1)$

Continued...

Question 5 [20 marks]

(a) A population has size 5000, a mean of 3.5, and a standard deviation of 0.8. For the following sample size, find the mean and standard deviation of the sampling distribution of the sample mean.

(i) Sample size of 400. [3 marks]

(ii) Sample size of 40. [3 marks]

(b) Given the population data 2, 4, 4, 6, and 8. Construct the sampling distribution of the sample mean for samples of size 2 by filling in the following two tables. Use subscript to differentiate the same data values, for example, 4_1 and 4_2. [10 + 4 marks]

No	Sample	Sample mean
⋮		

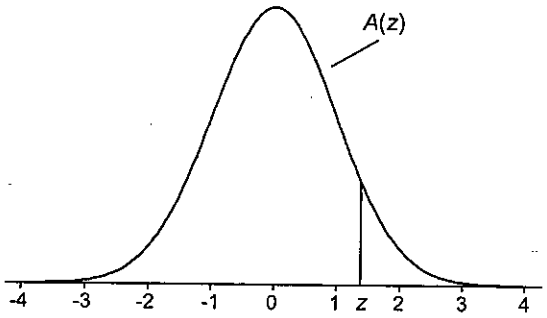
Sample mean	Frequency	Probability of sample mean
⋮		

Continued...

Appendix

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

End of Page